







# Examples of linear programs

An interpretation of the dual form

An application of duality






# An interpretation of the dual in a classic example

- A student wants to purchase a snack from a bakery to meet certain dietary requirements by choosing the best combination of **brownies** and **cheesecake**
- The characteristics of each product are given in the following table

Ingredients needed				
	3 oz	2 oz	2 oz	50 cts
	0 oz	4 oz	5 oz	80 cts
Requirements	6 oz	10 oz	8 oz	





# An interpretation of the dual in a classic example


- The student is following a diet trend which requires her to eat at least 6oz of chocolate, 8oz of cream cheese, and 10oz of sugar
- Her goal is to satisfy these requirements at minimal cost

Ingredients needed				
	3 oz	2 oz	2 oz	50 cts
	0 oz	4 oz	5 oz	80 cts
Requirements	6 oz	10 oz	8 oz	

The decision variables are

- $x_1$ : the amount of brownies that the student will purchase
- $x_2$ : the amount of cheesecake that the student will purchase

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The LP is







$$\min_{x_1, x_2} 50x_1 + 80x_2$$

$$\text{s.t. } 3x_1 \geq 6$$

$$2x_1 + 4x_2 \geq 10$$

$$2x_1 + 5x_2 \geq 8$$

$$x_1, x_2 \geq 0$$

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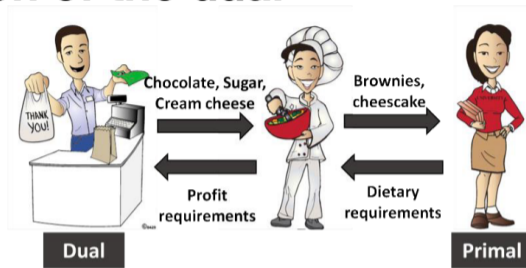
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The dual problem is given by

$$\begin{aligned}
 & \max_{y_1, y_2, y_3} \\
 & \text{s.t.} \quad \leq 0 \\
 & \quad \leq \\
 & \quad \leq
 \end{aligned}$$







# Interpretation of the dual



- Let's consider the problem from the point of view of a grocery store that provides the baker with the required ingredients
- We denote by
  - $y_1$ : the price of chocolate
  - $y_2$ : the price of sugar
  - $y_3$ : the price of cream cheese

The store wants to know how it should set them









Ingredients needed				
	3 oz	2 oz	2 oz	50 cts
	0 oz	4 oz	5 oz	80 cts
Requirements	6 oz	10 oz	8 oz	

the sum of the cost of the ingredients should not exceed the price of the product:

$$3y_1 + 2y_2 + 2y_3 \leq 50$$

$$4y_2 + 5y_3 \leq 80$$

Ingredients needed				
	3 oz	2 oz	2 oz	50 cts
	0 oz	4 oz	5 oz	80 cts
Requirements	6 oz	10 oz	8 oz	

The grocery store knows that the baker will buy at least 6oz of chocolate, 10oz of sugar, and 8oz of cream cheese to satisfy the student's requirements

Hence, it should set the prices of these ingredients to maximize profit:

$$\max_{y_1, y_2, y_3} 6y_1 + 10y_2 + 8y_3$$

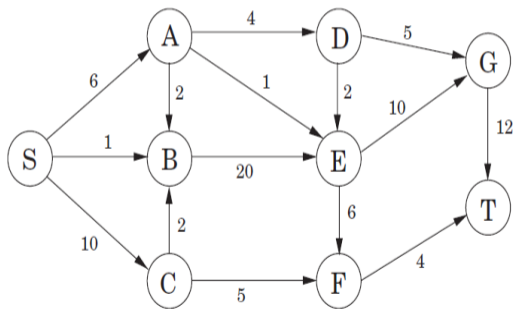
The optimization problem that the grocery store has to solve is the dual problem

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# Applications of duality

## Max flow/Min cut problem



$x_{SA}, x_{AD}, x_{BE}, \dots, x_{GT}$  ← Decision variables

max.  $x_{SA} + x_{SB} + x_{SC}$  ← Objective function

s.t.

$x_{SA}, x_{AD}, x_{BE}, \dots, x_{GT} \geq 0$

$x_{SA} \leq 6, x_{AB} \leq 2, x_{EG} \leq 10, \dots, x_{GT} \leq 12$

$x_{SA} = x_{AD} + x_{AB} + x_{AE}$

$x_{SC} = x_{CB} + x_{CF}$

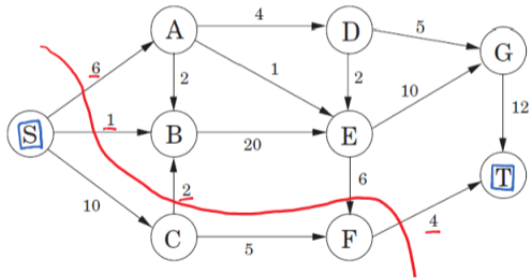
$\vdots$

$x_{CF} + x_{EF} = x_{FT}$

← Constraints

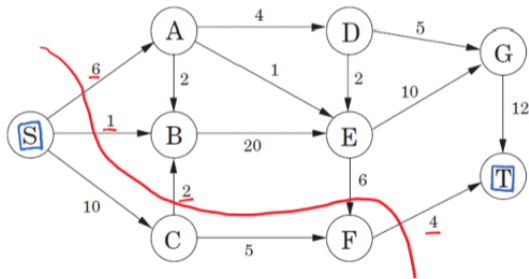
# Applications of duality

Max flow/**Min cut**.



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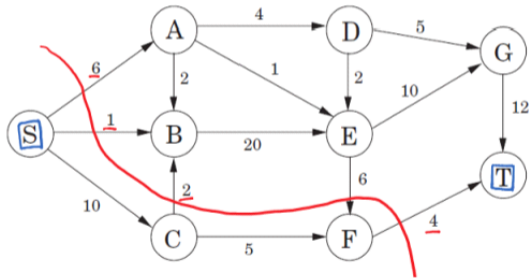
Max flow/**Min cut**.



- The dual of the max flow is the min cut problem

# Applications of duality

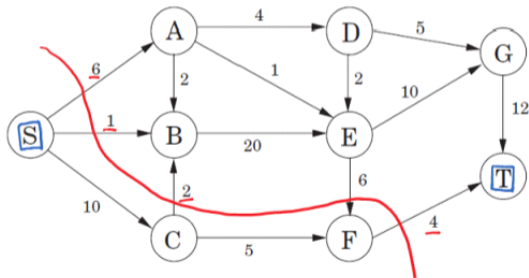
Max flow/**Min cut**.



- The dual of the max flow is the min cut problem
- The value of any min cut is an upper bound of the max flow (weak duality)

# Applications of duality

Max flow/**Min cut**.



- The dual of the max flow is the min cut problem
- The value of any min cut is an upper bound of the max flow (weak duality)
- There always exists a cut whose value matches the solution of the max flow (strong duality)