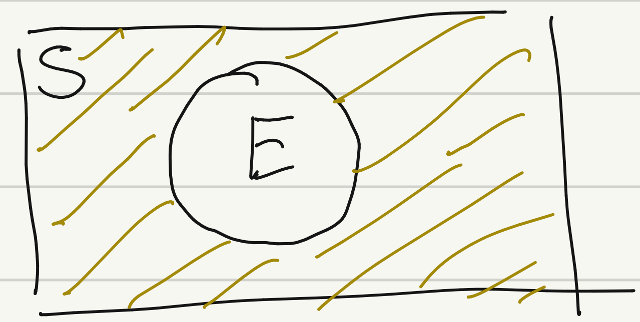
Complement of a set

Sample space S, event E

Complement of E, $E^c = S - E$



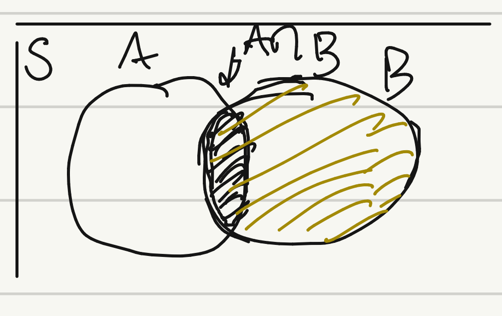
Event A, B:

$$(A \cup B)^c = A^c \cap B^c, \quad (A \cap B)^c = A^c \cup B^c$$

Conditional probability.

Probability of A conditioned on B

notation $\quad P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$



$$P(A \mid B) P(B) = P(A \cap B).$$

controlled environment

$$\frac{P(A \cap B \mid C)}{P(B \mid C)} = \frac{P(A \cap B \mid C) P(C)}{P(B \mid C) P(C)}$$

$$= \frac{P(A \cap B \cap C)}{P(B \cap C)}$$

$$= P(A \mid B \cap C)$$

Ex: 52-card deck, You draw four cards.

P(four aces) = ?

Events: A_1, A_2, A_3, A_4, $A_i = \{ i^{th}$ card is an ace$\}$

$P(A_1 \cap A_2 \cap A_3 \cap A_4)$

$= P(A_1 \cap A_2 \cap A_3 \mid \underset{\text{Background}}{A_4}) \cdot \underset{4/52}{P(A_4)}$

$= P(A_1 \cap A_2 \mid A_3 \cap A_4) \cdot \underset{\text{3 aces left out of 51 cards.}}{P(A_3 \mid A_4)} \cdot 4/52$

$= P(A_1 \mid A_2 \cap A_3 \cap A_4) P(A_2 \mid A_3 \cap A_4) \times 3/51 \times 4/52.$

$= 1/49 \times 2/50 \times 3/51 \times 4/52$

Ex: P(four aces | got four of a kind).

$= P(\text{number} = A) = 1/13.$

Ex Simpson's paradox.

Suppose we have new vs old drugs for curing a life-threatening disease.

	cured	died	cure rate
new	20	20	50%
old	24	16	60%

	Males		
	cured	died	cure rate
new	8	2	80%
drug	21	9	70%

	Females		
	cured	died	cure rate
new	12	18	40%
old	3	7	30%

$$\underset{0.5}{P(\text{cured} \mid \text{new})} = P(\text{cured} \cap \text{male} \mid \text{new}) + P(\text{cured} \cap \text{female} \mid \text{new})$$

$$= \overset{0.8}{P(\text{cured} \mid \text{male} \cap \text{new})} \cdot \overset{0.25}{P(\text{male} \mid \text{new})}$$

$$+ \overset{0.4}{P(\text{cured} \mid \text{female} \cap \text{new})} \cdot \overset{0.75}{P(\text{female} \mid \text{new})}$$

more weight on this

$$\underset{0.6}{P(\text{cured} \mid \text{old})} = \overset{0.7}{P(\text{cured} \mid \text{male} \cap \text{old})} \cdot \overset{0.75}{P(\text{male} \mid \text{old})}$$

$$+ \underset{0.3}{P(\text{cured} \mid \text{female} \cap \text{old})} \cdot \underset{0.25}{P(\text{female} \mid \text{old})}$$

new drug vs. old drug ?

see "causal inference".

Bayes' rule

suppose we know $P(A|B)$, but want to know $P(B|A)$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

sometimes unknown

$$= \frac{P(A|B)P(B)}{P(A \cap B) + P(A \cap B^c)}$$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)}$$

Ex Random testing on volunteers,
10% of which are drinkers.
The results are the following:
90% of the drinkers is positive ($+$).
20% of the non-drinkers is positive ($+$).

P(drinker | positive) , ND = non-drinkers.

(D = drinker, + = positive)

$$P(D \mid +) \overset{\text{Bayes' rule}}{=} \frac{P(+\mid D)\,P(D)}{P(+\mid D)P(D) + P(+\mid ND)\,P(ND)}$$

You need prob. of + given each group.

$$= \frac{0.9 \times 0.1}{0.9 \times 0.1 + 0.2 \times 0.9}$$

$$= \frac{0.09}{0.09 + 0.18} = \frac{0.09}{0.27} = \frac{1}{3}.$$