

Homework 4: October 8

Turn in your solutions for problem 1,2,3,6,7 and 8.

1. Show that if U has the chi-square distribution with m degrees of freedom, V has the chi-square distribution with n degrees of freedom, and U and V are independent, then $U + V$ has the chi-square distribution with $m + n$ degrees of freedom.
2. Let X_1, X_2, \dots, X_n be an iid sample from a distribution. Are $X_1 - \bar{X}, X_2 - \bar{X}, \dots, X_n - \bar{X}$ independent? Justify your answer.
3. Let X_1, \dots, X_{10} be a random sample from a normal population with mean $\mu = 2$ and variance $\sigma^2 = 1$.
 - Compute $\mathbb{P}(\bar{X} < 1)$.
 - Compute $\mathbb{P}(S^2 < 2)$.
4. Let $X_i \sim N(i, i^2)$ for $i = 1, 2, 3$. Use X_i 's to construct statistics with the chi-squared with 3 degrees of freedom
5. Let X_1, X_2, \dots, X_n be a random sample from a $n(\mu, \sigma^2)$ distribution. Calculate $\text{Var}(S^2)$. (Hint: Use the fact that $(n-1)S^2/\sigma^2 \sim \chi_{n-1}^2$.)
6. Let X be one observation from a $N(0, \sigma^2)$ population. Is $|X|$ a sufficient statistic? Please show your computations.
7. Assume $\sigma > 0$. Let X_1, X_2, \dots, X_n be a random sample from a pdf

$$f(x|\mu, \sigma) = \begin{cases} \frac{1}{\sigma} e^{-(x-\mu)/\sigma}, & \mu < x < \infty \\ 0, & \text{otherwise} \end{cases}.$$

Find a two-dimensional sufficient statistic for (μ, σ) . (Hint: To expand the sample pdf, you will need the function $1_{(\mu, \infty)}(\min_i x_i)$).

8. Let X_1, X_2, \dots, X_n be a random sample from the pdf

$$f(x|\theta) = \begin{cases} e^{-(x-\theta)} & \theta < x < \infty \\ 0 & \text{otherwise} \end{cases}.$$

Find a minimal sufficient statistic for θ .

9. Let X_1, X_2, \dots, X_n be a random sample from the pdf

$$f(x|\theta) = \frac{e^{-(x-\theta)}}{(1 + e^{-(x-\theta)^2})}.$$

Show that the statistic given by the *order statistic*:

$$T = T(X_1, X_2, \dots, X_n) = X_{(1)}, X_{(2)}, \dots, X_{(n)}$$

where $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$ is a minimal sufficient statistic for θ .