208891 Probabilistic Graphical Models **Spring 2019**

Lecture 18: March 13

Lecturer: Donlapark Pornnopparath

18.1 State space model

A *state space model* (SSM) is similar to HMM, but hidden states are now continuous. The most general form of this model is

$$z_t = g(u_t, z_{t-1}, \epsilon_t)$$
$$x_t = h(z_t, u_t, \delta_t),$$

where x_t is the observation, z_t is the hidden state, u_t us an optional input, h is the observation model, g is the transition model, δ_t is the observation noise and ϵ_t is the system noise.

A special case of SSM is when all the conditional distributions are Gaussians

$$p(z_t|z_{t-1}) = \mathcal{N}(z_t|A_t z_{t-1} + B_t u_t, Q_t)$$
$$p(x_t|z_t) = \mathcal{N}(x_t|C_t z_t + D_t u_t, R_t)$$

18.1.1 SSM for time series forecasting

We can model time series y_t with an SSM by adding three hidden components: level, trend and seasonality. The seasonality is a cycle of S time steps whose values sum to zero on average.

$$y_t = l_t + b_t + s_t + \epsilon_t^y \quad \text{(Forecast equation)}$$
$$l_t = l_{t-1} + b_{t-1} + \epsilon_t^y l \quad \text{(Level equation)}$$
$$b_t = b_{t-1} + \epsilon_t^b \quad \text{(Trend equation)}$$
$$s_t = -\sum_{i=1}^{S-1} s_{t-i} + \epsilon_t^s. \quad \text{(Seasonality equation)}$$